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Tax Optimization under Tax Evasion

The Role of Penalty Constraints

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This paper considers the problem of determining the optimal taxation for a group of individuals with random, independent and identically distributed incomes. Because a taxpayer's income is private information, it can only be verified through a costly audit. The purpose of this paper is to characterize the optimal tax schedule and auditing strategy that maximizes the government's net tax revenue under certain participation constraints that are related to the interests of the taxpayers. The authors determine the optimal evasion-proof strategy, depending on the penalty constraint. They show that under certain conditions, tax evasion may increase net tax revenue, but typically the optimal government strategy is evasion-proof.

Keywords: Russia, optimal tax schedule, penalty constraint, evasion-proof strategy.

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NON-TECHNICAL SUMMARY

In the short-term, the Russian economy confronts the following dilemma. On the one hand, there are important reasons (huge foreign debt and acute social problems) for increasing budgetary expenses. On the other hand, according to general opinion, tax rates should be reduced because it is impossible to work honestly under the present rates.

One possible solution to this dilemma relates to the fact that an essential part of the economy does not pay taxes now. According to estimates by the World Bank and IMF experts, the actual tax revenue in 1999 was less than 50% of the level corresponding to the honest behavior of taxpayers.

Reduction of tax rates by itself would not increase tax revenues. Those who do not pay would not pay, and those who pay would pay less. The only real way to solve this dilemma is by simultaneously optimizing the tax rates and audit strategy.

Our basic model considers a group of taxpayers with random, independent and equally distributed incomes. All taxpayers and the government know the distribution of income from the beginning of the tax period. At some time before paying tax, each taxpayer finds out her own income and sends a (not necessarily true) report to the government. The government does not know who has what actual income. It may audit a taxpayer in order to verify her reported income. An audit always determines a taxpayer's true income and has a fixed cost. The government collects taxes through the following mechanism. It sets a tax schedule dependent on reported income, a penalty schedule dependent on actual and reported incomes, and the probability of inspection dependent on individual's reported income.

Our purpose is to study the tax optimization problem in the "principal-agent" framework, that is, to find the government strategy that maximizes net tax revenue under the optimal behavior of agents (*i.e*. taxpayers) and the following participation constraints. The expected income of an agent should exceed a fixed desirable level. Under the worst possible outcome, the income of an agent should exceed a minimal level necessary for her "survival".

We consider the following variants of the penalty constraints: a) penalty is proportional to detected unpaid tax, b) net penalty (except the due tax) is proportional to hidden income; c) penalty is limited to the given minimal income of a taxpayer; d) penalty is proportional to detected hidden income.

Our main results are as follows. It is always optimal to set the maximal lump-sum tax under the above mentioned constraints. If the taxpayer's income variance is relatively small, then introducing any other additional tax does not increase net revenue. But if the income distribution is widely dispersed and the cost of an audit is relatively low, then the optimal evasion-proof strategy of the government is as follows. According to the optimal tax schedule, for all incomes below some threshold the after-tax income is equal to the minimal level specified by participation constraint, and for all higher incomes the tax schedule is flat. The optimal audit strategy is a known "cut-off" rule: reports below the threshold are audited with the minimal probability that makes tax evasion unprofitable and the rest of the reports are not audited. In some sense the optimal contract does not depend on the penalty rule: it does not matter if the penalty is proportional to the evaded tax or hidden income, or includes both components. The sum of the penalty coefficients determines the probability of an audit and the threshold level of income.

An important issue is whether the specified tax scheme is optimal in general, that is, the optimal tax policy of the government is always evasion-proof. In contrast to the conventional point of view, we found that the optimal government strategy permits tax evasion for some distributions under the mentioned penalty schemes a or b. This happens if there are two possible levels of income, its variance is neither very high, nor very low, and the penalty for evasion is sufficiently soft. However, for typical continuous distributions of income, the optimal evasion-proof contract is optimal in general (at least, if the penalty is proportional to the evaded tax). The same proposition is true for any distributions under constraints c or d.

1. INTRODUCTION

Informational asymmetries between the government and the taxpayer generate important constraints on the choice of tax policy. In much of the recent literature on optimal income taxation (Reinganum and Wilde, 1985; Border and Sobel, 1987; Chander and Wilde, 1998 and others), individual incomes are treated as exogenous. Informational constraints arise since an individual's income cannot be directly observed. It can only be verified through a costly audit. In this setting a government strategy must include, besides tax rates, an audit strategy and a scheme of penalties for misreporting income. This opens up interesting questions regarding the interaction of optimal tax rates, audit strategy, participation and penalty constraints.

Early optimal income tax results (see Atkinson, Stiglitz, 1980) primarily relate to the case where all taxpayers are risk-neutral, their participation constraints concern expected after-tax incomes, and the government knows the type of every agent, in particular, her probabilistic distribution of income. In this situation, informational asymmetry is not essential: according to the Welfare theorem (*ibid*.), the government can reach the first best result by means of a type-specific lump-sum tax, and an audit is unnecessary.

However, in practice taxpayers cannot be completely risk-neutral under any tax policy. For instance, for a firm there typically exists a threshold income (dependent on its type) such that, if after-tax income falls below this value, then the firm cannot obtain access to credit and becomes bankrupt. Previous models of optimal taxation under tax evasion (Chander and Wilde, 1998; Mookherjee and Png, 1989) take this condition into account in the form of a participation constraint. In this case, income after paying taxes and possibly fines is required to be non-negative in all circumstances, as if the firm became infinitely risk averse for outcomes below the threshold. The optimal tax in general depends on a taxpayer's income, and the determination of the optimal tax policy becomes a non-trivial task.

It should be noted that most of the literature on optimal tax enforcement either restricts attention to linear tax schedules or considers fixed taxes and penalties (see Cremer, Marchand and Pistieau, 1990; Sanchez and Sobel, 1993).

Mookherjee and Png (1989) study the tax enforcement problem in a contract theory-type setting. They consider risk averse taxpayers and include a moral hazard problem. Their model permits arbitrary tax and penalty schedules, subject to participation constraints. Their results show that such an approach has some disadvantages: the optimal penalty schedule is to fine a tax evader an amount equal to her entire income, irrespective of the amount of income that was concealed. Such a draconian rule is unavailable in practice. Moreover, as Chander and Wilde (1998, p. 177) mention, it is unrealistic to assume that penalties are a choice variable of the tax authorities. Actual penalties may be constrained, for instance, by the common ethical norm of letting the punishment fit the crime.

Chander and Wilde (1998) consider the tax optimization problem for risk neutral taxpayers with an exogenous income distribution. Proceeding from the previous argument, they consider another type of penalty constraint: the penalty payment after an audit is equal to the hidden income. They introduce the notion of an efficient scheme including tax, penalty and auditing probability functions such that any other scheme does not allow an increase in the expected payment of any taxpayer without increasing the probabilities of auditing for some reported income. They show that it is possible to restrict attention to evasion-proof schemes and to establish their general properties: the tax function is non-decreasing with a non-increasing average tax rate; audit probabilities are determined by marginal payment rates and are non-increasing.

They show that it is possible to restrict attention to evasion-proof schemes and to establish their general properties: the tax function is non-decreasing with a non-increasing average tax rate; audit probabilities are determined by marginal payment rates and are non-increasing.

Let us note that the tax optimization problem has been studied under penalty constraints that are not used in practice. Typically, identified tax evaders repay the detected unpaid tax and also pay some penalty. In several countries this penalty is proportional to the unpaid tax, in some others (including Russia) it is proportional to the hidden income. As far as we know, there are no prior results establishing the optimality of evasion-proof contracts given these constraints. Moreover, the previous literature establishes the general properties of the optimal tax schedules but does not provide a tool for their determination.

The present paper will study the tax optimization problem under various penalty constraints and determine the optimal tax schedules depending on the characteristics of the taxpayers. We will find under what conditions the optimal government strategy is evasion-proof, and alternatively, possible reasons for the government to permit tax evasion.

Our basic model includes the government and a group of taxpayers with identical but independent random distributions of incomes. The government (the principal) and each taxpayer (the agent) know the income distribution. At some time before paying tax, the agent finds out her own income. But the government does not know who has what income. An agent makes a report of her income to the principal, while the principal may audit an agent in order to verify her income. An audit always determines the taxpayer's actual income, and each audit involves a fixed cost. The government collects taxes through the following mechanism. It sets a tax schedule (or schedule of pre-audit payments) dependent on the reported income, and a penalty schedule (or post-audit payment schedule) dependent on the actual and reported incomes. The government also chooses the probability of inspection, dependent on the reported income.

Our purpose is to study the tax optimization problem in this principal-agent framework, that is, to find the government strategy that maximizes expected net tax revenue (net of auditing costs) under the optimal behavior of agents and the participation constraints related to the taxpayer's income. We assume that every taxpayer is risk-neutral and aims to maximize her expected after-tax income. We consider two types of participation constraints, one based on expected after-tax and penalty income, and the second based on realized after-tax and penalty income:

(1) The expected income of an agent under her optimal behavior should exceed a fixed level, called her "alternative income". An otherwise unconstrained expected revenue maximizing government would find it desirable to leave a taxpayer with her alternative income.

(2) Since the income of every agent is a random variable, we require that under optimal behavior and the worst random outcome, the actual (not expected) income of an agent should exceed a minimal value necessary for the "survival" of the taxpayer.

Further, we consider the following variants of permissible penalties imposed upon detected tax evaders: a) the penalty is proportional to the detected, unpaid tax; b) the "pure" penalty is proportional to the hidden income; c) the penalty is bounded because of the required minimal after-tax and penalty income of an agent; and, d) the payment after an audit is proportional to the detected hidden income.

Our main results are as follows. The optimal contract is always evasion-proof under the penalty constraints c and d. Some tax evasion may be optimal, however, for a net revenue maximizing government if there are two possible levels of income; the pure penalty is proportional to the unpaid tax or hidden income (penalty variants a and b) and the proportionality coefficient is sufficiently low. Among evasion-proof strategies the optimal tax is either equal to the entire income above the minimal level for small incomes and is flat for higher incomes, or it is flat for all incomes. In the former case, the optimal audit strategy is a probabilistic cut-off rule (*cf*. Sanchez and Sobel, 1993): every reported income below some threshold is audited with a probability that makes underreporting unprofitable, and every higher report is not audited. Under penalty constraint a and general assumptions on the density of the income distribution, the optimal evasion-proof strategy is optimal in general.

We generalize some of these results for an important extension of the basic model where the result of an audit depends not only on government effort but also depends on its ratio to the total and hidden income of the taxpayer.

The new contribution of our paper to the existing literature is as follows. Besides the penalty schemes c and d, studied by CW, we consider two other practically important variants and show that the optimal contract is not necessarily evasion-proof. We examine how this property depends on the penalty coefficients, minimum alternative income and other parameters of the model. We obtain transparent results on the optimal government strategies, including the explicit tax schedules and audit rules. We study a more general model where the result of an audit depends on the relationship between an auditor's effort and the real and reported income, and we generalize the results of CW on the optimality of evasion-proof contracts for this setting.

The major assumptions that limit the generality of our analysis are standard in the literature and are also used by CW. As in most of the literature, we rule out supply side effects of income taxation. We do not think that introducing supply effects would essentially change our results. As Mirrlees (1971) shows, these effects may only reinforce the regressivity of optimal tax schedules. Our optimal tax schedule, however, is already extremely regressive.

The other major assumption is that taxpayers are risk neutral. Since we focus on the taxation of firms and the participation constraint prevents tax bankruptcy, this condition seems to be not very restrictive.

The paper proceeds as follows. In Section 2.1 we define the basic model. Section 2.2 solves the tax optimization problem for the case of two possible levels of income and determines under what conditions tax evasion is profitable for the government. The optimal evasion-proof contract for different penalty constraints is determined in Section 2.3. In Section 3 we study an extension of the basic model where the result of an audit depends not only on the government effort but also on its ratio to the total and hidden income of the taxpayer. Section 3.2 shows that the optimal contract is still evasion-proof under constraints c or d. In

Section 3.3 we explore the general contract approach and show that there always exists an optimal simple contract under constraint c. Section 4 concludes with several remarks and policy implications related to the Russian economy. Technical proofs of several propositions are given in Appendix.

2. A BASIC MODEL

2.1. Model specification. The first best solution

We consider an interaction between the government and a group of taxpayers. The income I of each taxpayer is an independent random value with distribution function $G(I)$ concentrated in the interval $[I_L, I_H]$. The taxpayer's income is private information. A government strategy s_G, or simple contract, includes three components: a non-decreasing tax function $T(I_r)$ where $I_r(I) \in [I_L, I]$ is reported income, an audit probability $p(I_r) \in [0,1]$ and a penalty function $F(I, I_r)$ that determines the additional payment of the agent depending on her actual and reported incomes. In this version of the model, an audit always reveals the true income and its costs is fixed.

Under a given government strategy, each taxpayer aims to maximize her expected income. So, depending on her actual income, her reported income is

$$I_r(I, s_G) \to \min_{I_r}\{T(I_r) + p(I_r)F(I, I_r)\} \stackrel{def}{=} T_{eff}(I, s_G)\,. \tag{2.1.1}$$

The value on the right-hand side is the effective tax, or the expected total payment from an agent with income I under strategy s_G.

The problem of the government is to maximize expected tax revenue net of auditing costs[1]

$$\int_{I_L}^{I_H} \{T_{eff}(I, s_G) - cp[I_r(I, s_G)]\} dG(I) \to \max_{s_G} \tag{2.1.2}$$

[1] In general tax revenue maximization is not the main purpose of the government. Another typical setting is that the government aims to maximize a social welfare function dependent on the after-tax income of taxpayers and net tax revenue (*cf*. Atkinson, Stiglitz, 1980). Then, the optimal strategy coincides with the solution of our model under a certain value of alternative income.

(below we omit the limits of integration) under the following participation constraints:

$$\int [I - T_{eff}(I, s_G)]dG(I) \geq I_{alt} , \tag{2.1.3}$$

that is, the expected income of an agent under her optimal behavior should exceed her alternative income I_{alt}. The value I_{alt} is a desirable level of a taxpayer's expected income (from the point of view of the government) in our model. Another interpretation is that this is the expected income of a firm if it moves its activity to another country or to an untaxable field ("reservation" value of income). Note that a firm makes its choice at the beginning of the tax period when only distribution $G(I)$ (not concrete income I) is known.

$$T[I_r(I)] + F[I, I_r(I)] \leq I - I_{\min} \quad \text{if} \quad p[I_r(I)] > 0 , \tag{2.1.4a}$$

and

$$T[I_r(I)] \leq I - I_{\min} \quad \text{if} \quad p[I_r(I)] < 1 , \tag{2.1.4b}$$

that is, under optimal behavior and the worst random outcome, the income should exceed the value $I_{\min}$ which is necessary for the "survival" of a taxpayer. In practice $I_{\min}$ may be a threshold value such that if the rest on the firm's account falls below this value, then the firm cannot get credit and has to stop operating.

We consider several variants of penalty schemes:

a) $F(I, I_r) = (1 + \delta_a)[T(I) - T(I_r)]$ (penalty is proportional to detected unpaid tax);

b) $F(I, I_r) = T(I) - T(I_r) + \delta_b(I - I_r)$ (net penalty is proportional to hidden income);

c) $0 \leq F(I, I_r) \leq I - T(I_r) - \hat{I}$ for any I, I_r (penalty is bounded because of the given minimal income of an agent $\hat{I}(\leq I_{\min})$ under the non-optimal behavior of a taxpayer);

d) $F(I, I_r) = \delta_d(I - I_r)$ (the payment after audit is proportional to detected hidden income).

Actually the government can choose the penalty only under condition c. The first inequality in this constraint means that there are no premiums either for telling the truth or for lying. This condition rules out the possibility of making enforcement costs arbitrarily small by offering the agents a lottery where they get a large reward for telling the truth with small probability and thus inducing them to use strategies preferable for the government.[2]

The former two variants of penalty schemes correspond to the actual legislation in different countries while the latter two are studied by CW. Their results (see Section 1 and Lemma 3 on p.177) imply the following theorem.

Theorem 2.1.1. Under penalty constraints c or d, there exists an evasion-proof optimal government strategy for the problem (2.1.2 – 2.1.4), that is, such a solution s_G that $I_r(I, s_G) = I$ for any I. Moreover, the optimal penalty is always the maximal one in the case c: $F(I, I_r) = I - T(I_r) - \hat{I}$ for any $I \neq I_r$.

Note. CW consider only the case $\delta_d = 1$ for variant d, but their proof holds true for any $\delta_d > 0$.

Section 2.2 below proves a similar result for the more general model of an audit. The next section shows that the theorem is not true under penalty constraints a or b when a certain level of tax evasion may be optimal for the government.

The two values play a crucial role in the subsequent analysis of the model.

$$T_{LM} \overset{def}{=} I_L - I_{\min}$$

is the maximal possible tax on the lower income under the participation constraint (2.1.4). This value characterizes the stability of a taxpayer under the worst state of nature.

$$\Delta EI \overset{def}{=} \int I dG(I) - I_{alt}$$

is an expected before-tax surplus to an agent with respect to her alternative income. For a firm, this value characterizes the profitability of production.

[2] Tirole (1992) describes the corresponding contract in the general principal-agent setting. CW do not require the specified inequality but assume only that there are no large premiums for telling the truth. However, it is easy to see that large premiums for a small lie may have the same effect. Moreover, we do not consider even limited premiums since they create strong incentives for collusion between the tax service and taxpayers.

If we exclude the individual incentive constraint (2.1.1), assuming that taxpayers do not evade, and set $p(I_r) \equiv 0$, then we obtain the problem for the first best solution of revenue maximization:

$$R[T(.)] = \int T(I)dG(I) \to \max_{T(I)} \tag{2.1.5}$$

under constraints

$$\int T(I)dG(I) \le \Delta EI\ , \tag{2.1.6}$$

$$T(I) \le I - I_{\min},\ \ I \in \left[I_L, I_H\right]. \tag{2.1.7}$$

Obviously, this solution does not depend on the penalty constraint. Consider the following expression for the tax function:

$$T(I) = T(I_L) + \Delta T(I),$$

where $T(I_L)$ is a lump sum component paid by each taxpayer and $\Delta T(I) \ge 0$ is an additional tax dependent on the income. Proceeding from (2.1.7), $T(I_L) \le T_{LM}$.

Proposition 2.1.2. The first best net tax revenue is equal to ΔEI. If $T_{LM} \ge \Delta EI$, then the optimal government strategy for the original problem (2.1.2 – 2.1.4) corresponds to the first best solution: $T(I) \equiv \Delta EI$, $p(I) \equiv 0$.

Proof. According to constraint (2.1.7), $R(I) \le \Delta EI$. If $T_{LM} \ge \Delta EI$, then the given strategy provides revenue ΔEI and meets all constraints. Now consider the case $T_{LM} < \Delta EI$. Let us find the first best solution. Consider a function

$$R(\overline{I}) = \int_{I_L}^{\overline{I}} (I - I_{\min})dG(I) + (\overline{I} - I_{\min})[1 - G(\overline{I})]$$

that determines the revenue for the tax schedule

$$T(I) = \begin{cases} I - I_{\min}, & \text{if } I \le \overline{I}, \\ \overline{I} - I_{\min}, & \text{if } I > \overline{I}. \end{cases}$$

Note that $R(\overline{I})$ is continuous in $\overline{I}$ and

$$R(I_L) = T_{LM} < \Delta EI,\ \ R(I_H) = \int IdG(I) - I_{\min} > \Delta EI\ .$$

Hence, there exists a value $\bar{I}$ such that $R(\bar{I}) = \Delta EI$.

Thus, whenever $T_{LM} \geq \Delta EI$, the government can get the first best result by means of the lump sum tax $T(I) \equiv T_L = \Delta EI$. It is unnecessary to collect any other taxes and organize audits. Proceeding from the previous discussion of conditions (2.1.3), (2.1.4), the inequality shows that the expected surplus is less than the maximal lump-sum payoff that does not undermine the activity of a firm under unfavorable conditions. The case $T_{LM} < \Delta EI$ is more sophisticated. The next section solves the problem for one type of income distributions and shows that, under certain conditions, the government can get the maximal revenue only if taxpayers evade.

2.2. A model with two possible levels of income

Consider a group of taxpayers who earn a high income I_H with probability q and a low income I_L with probability $1-q$. A government strategy includes taxes T_L and T_H on these incomes, a probability p of auditing low income reports and a penalty F on tax evasion (if this penalty is not given exogenously). Denote

$$\Delta I \overset{def}{=} I_H - I_L, \quad \Delta T \overset{def}{=} T_H - T_L \geq 0.$$

A taxpayer's strategy is her report $I_r \in \{I_L, I_H\}$ that she sends if she earns a high income. Since a taxpayer maximizes her expected income,

$$I_r = I_L \quad \text{if} \quad pF < \Delta T, \quad \text{otherwise} \quad I_r = I_H. \tag{2.2.1}$$

The government aims to maximize expected net tax revenue, so the formal problem is to find

$$R \to \max_{(T_L, \Delta T, F, p)}, \tag{2.2.2}$$

where $R = T_L + q\Delta T - p(1-q)c$ if $pF \geq \Delta T$, otherwise $R = T_L + qpF - pc$. Participation constraints take the form of

$$T_L + q\min(\Delta T, pF) \leq \Delta EI = I_L + q\Delta I - I_{alt}, \tag{2.2.3}$$

$$T_L \leq T_{LM} = I_L - I_{\min}; \quad I_H - T_H \geq I_{\min} \text{ if } I_r = I_H, \tag{2.2.4}$$

otherwise $I_H - T_L - F \geq I_{\min}$.

The four variants of the penalty constraints in this model look like

a) $F = (1 + \delta_a)\Delta T$;

b) $F = \Delta T + \delta_b \Delta I$;

c) $I_L + \Delta I - T_L - \max(\Delta T, F) \geq \hat{I}$;

d) $F = \delta_d \Delta I$.

If $T_{LM} \geq \Delta EI$, that is equivalent to $I_{alt} - I_{\min} \geq q\Delta I$, then, according to Proposition 2.1.2, optimal net revenue $R^* = \Delta EI$ is the same for problem (2.1.1) – (2.1.4) with any penalty constraint a – d and coincides with the first best solution. The optimal government strategy is $\Delta T = 0$, $T_L = \Delta EI$, $p = 0$.

If $T_{LM} < \Delta EI$, then the first best revenue value is the same but requires a combination of the both kinds of taxes. Let us find solutions of the tax optimization problem (2.2.1) – (2.2.4) for different penalty constraints.

a) $F_a = (1 + \delta_a)\Delta T$. Note that taxpayers evade in this case if

$$\Delta T > 0,\ p < p_a^* \stackrel{def}{=} 1/(1 + \delta_a) .$$

The proposition below shows how the optimal government strategy depends on the parameters of the model.

Proposition 2.2.1. First, consider the case where

$$c < q\Delta I . \tag{2.2.5}$$

Then, for any fixed c, q, ΔI and δ_a , there exist three variants of the optimal government strategy dependent on the difference $I_{alt} - I_{\min}$. If

$$q\Delta I \leq I_{alt} - I_{\min} , \tag{2.2.6}$$

then the optimal strategy is to set lump-sum tax $T_L = \Delta EI (= I_L + q\Delta I - I_{alt})$, $\Delta T = 0,\ p = 0$.

In the interval

$$q\Delta I \left(1 - \frac{1 - q}{1 + \delta_a} \right) < I_{alt} - I_{\min} < q\Delta I \tag{2.2.7}$$

the optimal strategy includes the following: $T_L = T_{LM}$, the additional tax $\Delta T = \Delta I p_a^*$ (hence, $F = \Delta I$), the audit probability

$$p = \frac{\Delta EI - T_{LM}}{q\Delta I} \quad \left(= 1 - \frac{I_{alt} - I_{\min}}{q\Delta I} \right) < p_a^* .$$

The gross revenue R_g is ΔEI (that is maximal under (2.2.3)), and the net revenue $R = \Delta EI - pc$.

In the remaining interval

$$0 < I_{alt} - I_{\min} \le q\Delta I \left(1 - \frac{1-q}{1+\delta_a} \right), \tag{2.2.8}$$

the optimal strategy is $T_L = T_{LM}$, $p = p_a^*$, $\Delta T = (\Delta EI - T_{LM})/q$. Honest behavior of taxpayers is optimal, and the gross revenue R_g is again ΔEI while the net revenue $R = \Delta EI - (1-q)p_a^* c$.

So in the interval (2.2.7), it is optimal for the government to permit tax evasion of taxpayers with high incomes and to get revenue through penalties. The costs of such collection turns out to be less than the cost of the optimal evasion-proof strategy (that provides the same gross revenue).

Fig. 1 summarizes our results on the optimal government strategy under condition (2.2.5).

If $c(1-q)p_a^* < q\Delta I \le c$, then the optimal government strategy corresponds to Fig. 2. In the interval

$$q\Delta I - c(1-q)p_a^* \le I_{alt} - I_{\min} < q\Delta I ,$$

the optimal strategy is $(T_L = T_{LM},\ p = 0,\ \Delta T = 0)$. The gross and net revenue under this strategy is $R = T_{LM} < \Delta EI$.

In the interval

$$0 \le I_{alt} - I_{\min} < q\Delta I - c(1-q)p_a^* ,$$

the optimal strategy is ($T_L = T_{LM}$, $p = p_a^*$, $\Delta T = (\Delta EI - T_{LM})/q$), as in the interval (2.2.8) under (2.2.5).

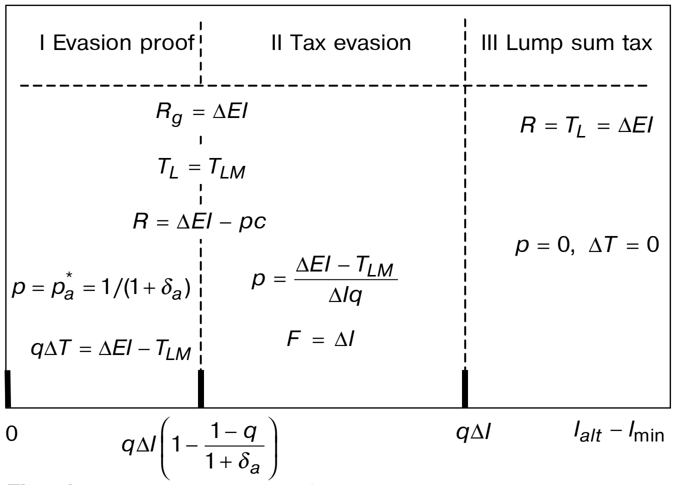

Fig. 1. The optimal tax enforcement strategy under penalty constraint $F_a = (1+\delta_a)\Delta T$ and condition $c < q\Delta I$.

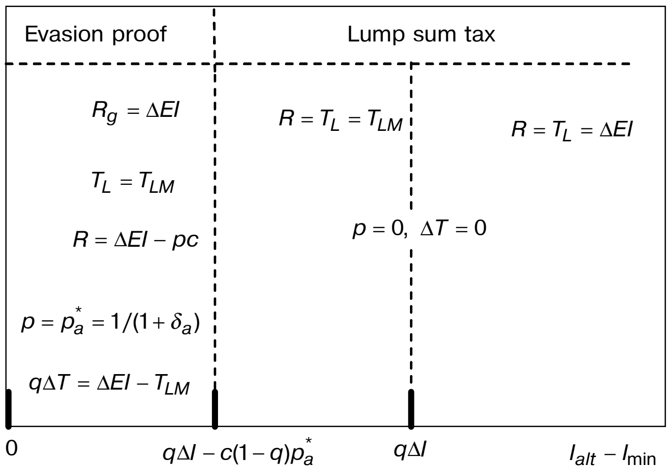

Fig. 2. The optimal tax enforcement strategy under penalty constraint a) and condition $c(1-q)p_a^* < q\Delta I \le c$.

Finally, under $q\Delta I \le c(1-q)p_a^*$, lump-sum taxation always provides the maximal revenue. If $I_{alt} - I_{\min} < q\Delta I$, then the optimal strategy is $(T_L = T_{LM},\ p = 0,\ \Delta T = 0)$.

Let us discuss this result. Return to the case $c < q\Delta I$. Since ΔI characterizes the variance of the income distribution (under a fixed q), we may say that lump-sum taxation is optimal when the variance is sufficiently small with respect to the difference between I_{alt} and $I_{\min}$. If the variance is relatively large, then the optimal strategy is evasion-proof, and in the intermediate interval, the optimal government strategy permits tax evasion. Another interpretation relates to the values ΔEI and T_{LM}.

Note that inequalities (2.2.6 – 2.2.8) are equivalent to

$$0 \le T_{LM} - \Delta EI \,,$$

$$0 < \Delta EI - T_{LM} < q\Delta I \frac{1-q}{1+\delta_a} \,,$$

$$q\Delta I \left(\frac{1-q}{1+\delta_a} \right) \le \Delta EI - T_{LM} < q\Delta I \,,$$

respectively. Since the difference $\Delta EI - T_{LM}$ characterizes the profitability of production versus its stability, we may conclude that lump-sum taxation is optimal when profitability is relatively low (area (2.2.6)), the monotone tax and the audit strategy enforcing honest reporting are optimal where profitability is relatively high (2.2.8), and the monotone tax and the audit strategy permitting tax evasion are optimal in the intermediate area (2.2.7). Fig. 2 shows that if the cost of an audit is relatively high $(c \ge q\Delta I)$, then lump-sum taxation is more profitable than allowing tax evasion in the intermediate area. In other respects the picture is similar to the previous case.

The intuition for this result is as follows.

The whole set of government strategies divides into three subsets: area I, where $pF \ge \Delta T > 0$ and honest reporting is optimal for taxpayers; area II, where $\Delta T > pF > 0$ and it is optimal to evade paying taxes; and area III, where $\Delta T = 0$. In the latter area, the optimal auditing rule is obvious: $p = 0$ and the maximal revenue under $T_{LM} < \Delta EI$ is T_{LM} irrespective of the penalty constraint.

In areas I and II the optimal strategy includes the maximal possible lump-sum tax $T_L = T_{LM}$ (its collection does not require any audit costs). In area II the revenue is collected in the form of penalties. The maximal gross revenue is ΔEI. In order to get it, we set $pF = (\Delta EI - T_{LM}) / q$. Under this condition, the penalty should be as large as possible, since we aim to minimize audit costs. Because of participation constraint (2.2.4), $F_{\max} = \Delta I$. Condition (2.2.5) shows that collection of penalties is profitable under such fine.

In area I the revenue comes from taxes. By similar argumentation, $\Delta T = (\Delta EI - T_{LM})/q$, and the minimal audit probability that makes honest reporting optimal is p_a^*. By comparing p_a^* with the optimal audit probability in area II, we determine the border between areas II and III. In other cases, the reasoning is similar. See Appendix for the detailed proof.

Now consider constraint b) $F = \Delta T + \delta_b \Delta I$.

Proposition 2.2.2. Let $c < q\Delta I$, $q + \delta_b < 1$. Then in the area

$$q\Delta I(q + \delta_b) < I_{alt} - I_{\min} < q\Delta I$$

the optimal strategy includes: $T_L = T_{LM}$, the additional tax $\Delta T = \Delta I(1 - \delta_b)$ (hence, $F = \Delta I$), the audit probability

$$p = \frac{\Delta EI - T_{LM}}{q\Delta I} \quad \left(= 1 - \frac{I_{alt} - I_{\min}}{q\Delta I} \right) < \Delta T / F\ ,$$

so this strategy permits tax evasion. The gross revenue R_g is ΔEI, and the net revenue $R = \Delta EI - pc$.

In the area

$$0 < I_{alt} - I_{\min} \le q\Delta I(q + \delta_b)$$

the optimal strategy is

$$T_L = T_{LM}, \quad \Delta T = (\Delta EI - T_{LM})/q\ , \quad p_b^* = \Delta T / (\Delta T + \delta_b \Delta I)\ . \qquad (*)$$

Honest behavior of taxpayers is optimal, and the gross revenue R_g is again ΔEI while the net revenue $R = \Delta EI - (1 - q)p_b^* c$. For any other values of the parameters, either strategy (*) or the lump-sum tax ($T_L = T_{LM}$, $p = 0$, $\Delta T = 0$) are optimal.

Proof is given in Appendix. Note that, in contrast to case a, if the penalty coefficient δ_b is sufficiently large ($> 1 - q$), then the optimal contract is always evasion-proof.

Thus, for certain parameter values, the optimal contract implies tax evasion under constraints a or b. This is in contrast to cases c and d, where an optimal contract, which is evasion-proof, always exists according to Theorem 2.1.1.

Let us specify the optimal strategies. Consider the problem under c) $F \le I_H - \hat{I} - T_L$.

Proposition 2.2.3. In the area $\Delta EI > T_{LM}$, the optimal government strategy is

$$(T_L = T_{LM},\ \Delta T = (\Delta EI - T_{LM})/q,\ F = F_c \overset{def}{=} I_H - \hat{I} - T_{LM},$$

$$p = p_c \overset{def}{=} \Delta T / F_c) \text{ if } qF_c > (1-q)c .$$

Then the gross revenue R_g is ΔEI while the net revenue $R = \Delta EI - (1-q)p_c c$. Under $qF_c \le (1-q)c$, the lump-sum tax $(T_L = T_{LM},$ $p = 0,\ \Delta T = 0)$ is optimal for the government.

Proposition 2.2.4. Under penalty constraint d) $F_d = \delta_d \Delta I$, the optimal government strategy in the area $\Delta EI \ge T_{LM}$ is

$$(T_L = T_{LM},\quad \Delta T = (\Delta EI - T_{LM})/q,\quad p = p_d \overset{def}{=} \Delta T / F_d) \text{ if } qF_d > (1-q)c .$$

Then the gross revenue R_g is ΔEI while the net revenue $R = \Delta EI - (1-q)p_d c$. Otherwise, the lump-sum tax $(T_L = T_{LM}, p = 0,$ $\Delta T = 0)$ is optimal for the government.

See Appendix for the proofs.

Now, let us compare the penalty constraints from the point of view of the government.

Under any constraint a, b or d, the maximal revenue increases (or, at least does not decrease) in the penalty coefficient δ_a, δ_b or δ_d respectively, and tends to the first best value ΔEI as the coefficient tends to infinity. Under constraint c, the same proposition holds if we set $\delta_c = -\hat{I}$. So the most reasonable way to compare the penalty constraints is to find the equivalent values of the penalty coefficients. According to Propositions 2.2.1 – 2.2.4, we should consider several areas of parameters of the model. Let us start with $I_{alt} - I_{\min} < q\Delta I$,

$$\delta_a \geq \max\{0, \delta_a^*\} \text{ where } \delta_a^* \overset{def}{=} \frac{I_{alt} - I_{\min} - q^2 \Delta I}{q\Delta I - I_{alt} + I_{\min}}, \qquad (2.2.10)$$

δ_a^* is a solution of the equation

$$I_{alt} - I_{\min} = q\Delta I\left(1 - \frac{1-q}{1+\delta}\right).$$

Note that under $I_{alt} - I_{\min} < q^2 \Delta I$ such a solution is negative.

Let R_x^N denote the maximal net revenue in strategy set $N \in \{I, II, III\}$ under constraint $x \in \{a, b, c, d\}$. Then $R_a^I \geq R_a^{II}$ under (2.2.10) by Proposition 2.2.1. The optimal strategy in area I is

$$\Delta T = \Delta T^* \overset{def}{=} \frac{\Delta EI - T_{LM}}{q}, \; T = T_{LM}, \; p = \frac{1}{1+\delta_a}, \qquad (2.2.11)$$

and the penalty for evasion is $F = \Delta T^*(1+\delta_a)$. Let us determine $\delta_b, \widehat{I}$ and δ_d from the following equations:

$$F = \Delta T^*(1+\delta_a) = \Delta T^* + \Delta I \delta_b = \Delta I + I_{\min} - \widehat{I} = \Delta I \delta_d \, . \qquad (2.2.12)$$

Then for any penalty constraint, strategy (2.2.11) is optimal in area I. Government revenue and the incomes of all agents do not depend on the penalty constraint under (2.2.11 – 2.2.12). Since the optimal strategy in area III (where $\Delta T = 0$) and R_x^{III} do not depend on the penalty constraint, the same proposition is true for the globally optimal strategy. Thus, relation (2.2.12) determines the equivalent penalty coefficients under (2.2.11).

Now consider $I_{alt} - I_{\min} > q^2 \Delta I, \;\; \delta_a \in (0, \delta_a^*)$. Then $R_a^{II} > R_a^I$, and the optimal strategy in area II is

$$\Delta T = \frac{\Delta I}{1+\delta_a} \text{ (so } F = \Delta I \text{)}, \; T_L = T_{LM}, \; p = \frac{\Delta EI - T_{LM}}{q\Delta I}.$$

Note that government revenue and income distributions of taxpayers do not depend on δ_a since the agents evade taxes. Thus, all values $\delta_a \in \{0, \delta_a^*\}$ are equivalent in this sense. The only component that changes is a nominal tax rate ΔT.

It is easy to check that $\Delta T^*(1+\delta_a^*) = \Delta I(1-q)$. Equations (2.2.12) determine equivalent values $\delta_b^*, \hat{I}^*$ and δ_d^* for δ_a^*:

$$\delta_b^* = \frac{\Delta T^* \delta_a^*}{\Delta I} = \frac{I_{alt} - I_{\min} - q^2 \Delta I}{q \Delta I},$$

$$\delta_d^* = \frac{\Delta T^*}{\Delta I}(1+\delta_a^*) = 1 - q\ ,$$

$$\hat{I}^* - I_{\min} = q\Delta I\ .$$

But $\hat{I} \le I_{\min}$, so the equivalent value $\hat{I}^*$ exists only for

$$\delta_a \ge \bar{\delta}_a \overset{def}{=} \frac{\Delta I}{\Delta T^*} - 1.$$

Note that $F(\bar{\delta}_a) = \Delta I$.

Under constraint b, $R_b^{II} > R_b^I$ if $0 < \delta_b < \delta_b^*$ and $I_{alt} - I_{\min} > q^2 \Delta I$. The optimal strategy in II is the same, except for $\Delta T = \Delta I(1-\delta_b)$ (such that $F = \Delta I$), and brings the same incomes, gross and net revenues. All $\delta_b \in (0, \delta_b^*)$ and equivalent as well as $\delta_a \in (0, \delta_a^*)$.

Under constraint d, $R_d^I \ge R_d^{II}$, and for any $\delta_d < \delta_d^*$

$$R_d^I(\delta_d) < R_d^I(\delta_d^*) = R_a^I(\delta_a^*) = R_a^{II}(\delta_a)$$

for $\delta_a \in (0, \delta_a^*)$. Thus, in contrast to cases a and b, net revenue decreases together with δ_d until it reaches $T_{LM} = R_d^{III}$.

2.3 The optimal evasion-proof strategies

Now we return to the general model with income distribution $G(I)$. According to Theorem 2.1.1, the optimal contract is always evasion-proof under penalty constraints c and d. Proceeding from Propositions 2.2.1 – 2.2.2, we may assume that the same is true under constraints a or b if the penalty coefficients are sufficiently large. This section finds the optimal evasion-proof strategy for problem (2.1.2 – 2.1.4) under the following penalty constraint that generalizes a, b and d:

$$F = k\Delta T \; + l\Delta I \text{ , where } k, l \geq 0 \text{ , } \Delta I = I - I_r \text{ ; } \Delta T = T(I) - T(I_r) \text{ ;} \qquad (2.3.1)$$

note that a corresponds to $l = 0$, b — to $k = 1$, and d — to $k = 0$. Moreover, we show that this strategy is optimal in general under certain assumptions.

For any tax schedule T let $\overline{I}$ — denote such minimal income that $T(I) = T(\overline{I})$ for any $I > \overline{I}$, that is, the tax is flat for greater incomes. Then strategy $s_G = (T, p)$ is evasion-proof under (2.3.1) if and only if

$$p(I_r) \geq \frac{\Delta T}{F} \text{ for any } I > I_r,\ I_r < \overline{I} \text{ .} \qquad (2.3.2)$$

So the tax optimization problem for evasion-proof strategies is as follows:

$$[T(.), p(.)] \to \max\{R_g[T(.)] - C[p(.)]\} \text{ ,} \qquad (2.3.3)$$

where

$$R_g[T(.)] = \int T(I)dG(I)$$

is gross revenue,

$$C[p(.)] = c\int p(I)dG(I)$$

is the total audit cost, the government strategy meets condition (2.3.2) and participation constraints

$$R_g[T(.)] \leq \Delta EI = \int IdG(I) - I_{alt}, \qquad (2.3.4)$$

$$0 \leq T(I) \leq I - I_{\min} \text{ .} \qquad (2.3.5)$$

The following propositions characterize the optimal tax schedule and audit rule for problem (2.3.1 – 2.3.5).

Proposition 2.3.1. The optimal tax schedule T is concave, that is, $T(\lambda I_1 + (1-\lambda)I_2) \geq \lambda T(I_1) + (1-\lambda)T(I_2)$ for any $I_1 < I_2$, $\lambda \in [0,1]$. For any concave tax schedule, the optimal audit probability is

$$p(I_r, T) = (k + l\,/\,T_+^{'}(I_r))^{-1}, \qquad (2.3.6)$$

where $T_+^{'}(I_r)$ is the marginal tax rate for income I_r.

Proof. Note that gross revenue R_g does not depend on the audit rule in this case. For any tax schedule T, the optimal evasion-proof audit rule that minimizes the audit cost is

$$p(I) = \sup_{\Delta I > 0} [\Delta T / (k\Delta T + l\Delta I)]$$

for any

$$I < \overline{I}\,,\ p(I) = 0 \text{ for any } I \geq \overline{I}\,. \tag{2.3.7}$$

So the optimal probability of an audit of any report I_r is equal to the minimum that makes reporting I_r unprofitable for any $I > I_r$.

Assume from the contrary that T is not concave. Then its linear approximation in some interval lies above T (see Fig. 3). Let us change T for its linear approximation in this interval and denote the new tax schedule by T^*; then, gross revenue will increase. Proceeding from (2.3.7), the optimal probability will not increase for any I_r because

$$\sup \frac{\Delta T}{\Delta I} = \sup \frac{\Delta T^*}{\Delta I}$$

for any $I \leq I_1$ or $I \geq I_2$, and

$$\sup \frac{\Delta T}{\Delta I} \geq \sup \frac{\Delta T^*}{\Delta I}$$

for any $I \in (I_1, I_2)$. The only problem is that $R_g(T^*)$ may not meet constraint (2.3.4).

For any schedule T and tax level $Y \in [0, T(I_H)]$, let T_Y denote the following tax schedule: $T_Y(I) = T(I)$ if $T(I) \leq Y$, otherwise $T(I) = Y$. Note that $R_g(T_Y)$ continuously changes from 0 to $R_g(T)$ while Y changes from 0 till $T(I_H)$. The optimal audit probability $p(I_r, T_Y)$ does not decrease in Y for any I_r. So, if $R_g(T^*)$ exceeds ΔEI, then we can choose $Y < T(I_H)$ such that $R_g(T_Y) = \Delta EI$, and reduce audit costs. Hence T is not optimal. Finally, for any concave tax schedule, the ratio $\Delta T / \Delta I$ does not increase in ΔI, so we obtain expression (2.3.6) from (2.3.7) as ΔI tends to 0.

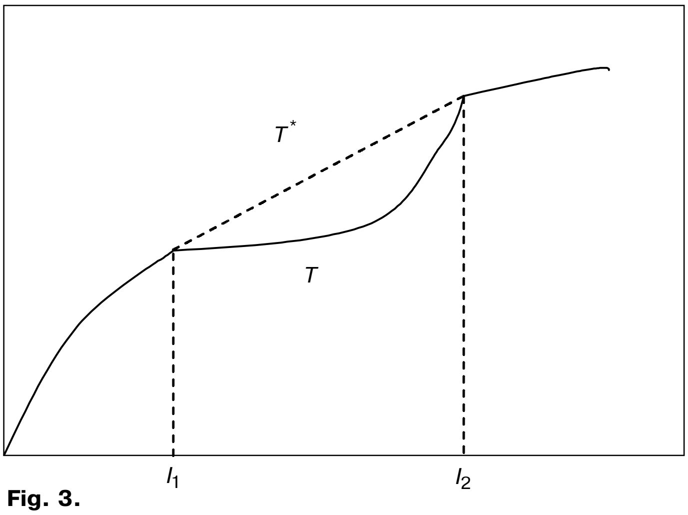

Fig. 3.

Note. CW obtained similar results under the penalty constraint $F = \Delta I$. This proposition also holds if the penalty is progressive in ΔI, that is, k depends on ΔI and increases in it.

Let there exist a density $\rho(I) = dG(I)/dI$ of the income distribution and $\mu(I) = (1 - G(I))/\rho(I)$ denote its hazardous rate. For typical statistic distributions, such as lognormal, uniform etc., this rate decreases in I. The following theorem finds the optimal evasion-proof government strategy for any income distribution with a decreasing hazardous rate. If $\Delta EI \le T_{LM}$ then, according to Proposition 2.1.2, the optimal strategy is lump-sum tax $T \equiv \Delta EI$. Now let $\Delta EI > T_{LM}$.

Theorem 2.3.2. If $\mu(I)$ decreases in I, then the optimal tax schedule T for problem (2.3.1 – 2.3.5) meets the following conditions: there exists $\bar{I}$ such that $T(I) = I - I_{\min}$ for any $I \le \bar{I}$ and $T(I) = T(\bar{I})$ for any $I > \bar{I}$. The optimal threshold $\bar{I}$ meets either conditions $R_g(T) = \Delta EI$ and

$$\mu(\bar{I}) > \frac{c}{k+l}, \text{ or } R_g(T) < \Delta EI \text{ and } \mu(\bar{I}) = \frac{c}{k+l}.$$

The optimal audit rule is $p(I) \equiv 1/(k+l)$ for $I < \bar{I}$, $p(I) \equiv 0$ for $I \ge \bar{I}$.

Moreover, if the penalty is proportional to the unpaid tax (penalty constraint a: $k = 1+\delta_a, l = 0$), the specified strategy is optimal in general for initial problem (2.1.2 – 2.1.4).

See Appendix for the proof.

The intuition is that, under the optimal audit rule, net tax revenue monotonously depends on marginal tax rates $T_+^{'}(I)$, so the optimal rate is either the maximal $(=1)$ or the minimal $(=0)$. Since T is concave by Proposition 2.3.1, it corresponds to Fig. 4.

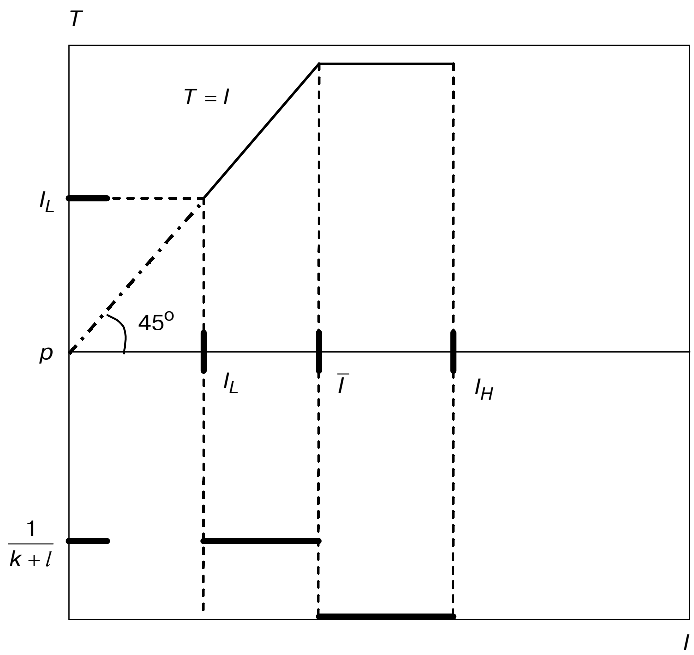

Fig. 4. The optimal evasion proof strategy under $I_{\min} = 0$, $\Delta EI > T_{LM}$ $(= I_L)$.

Thus, the optimal tax takes the whole income above the minimum for $I < \bar{I}$ and is flat for incomes above $\bar{I}$. The optimal audit strategy is a known cut-off rule (see Sanchez, Sobel, 1993): reports below threshold $\bar{I}$ are audited with the minimal probability that makes tax evasion unprofitable, and reports above $\bar{I}$ are not audited.

We conclude this section by comparing different penalty constraints. Theorem 2.3.2 shows that the optimal evasion-proof strategy and net revenue do not depend on the ratio of penalty coefficients k and l. The penalty proportional to the unpaid tax is equivalent to the penalty proportional to the hidden income since the optimal tax is equal to the income above the minimal level. Expenses for auditing decrease and net revenue increases in the sum $k+l$.

3. A MORE GENERAL MODEL OF AN AUDIT

3.1. Model specification

The standard audit model that we followed in Section 2 assumes that there are only two possible outcomes of the interaction between a taxpayer and the tax service: either there is no audit and no detected evasion, or an audit detects the whole hidden income at a fixed cost. However, this assumption typically does not hold in practice. Consider the following examples.

1. Let a taxpayer get her income from the trade of some good. Every unit of the good brings a fixed income i before tax. The volume of sales is a random value with a given distribution. For technical convenience let $i=1$. Then distribution $G(I)$ is the same for the income and for the volume of sales.

In order to evade paying tax, a taxpayer registers only some part I_r of her sales. The rest is sold for cash without registration. In order to detect tax evasion, auditors secretly observe all sales within some period and then check whether the sold production was registered. Their effort e is proportional to the checked production volume. If unregistered sales are randomly distributed in the whole set of sales, then the probability of detecting unregistered income d depends on the total and registered volumes and the auditors' effort as follows: $p(d \mid I, I_r, e) = C^{d}_{I-I_r} C^{e-d}_{I_r} / C^{e}_{I}$,

where $C^{e}_{I} = e!(I-e)!/I!$ is a binomial coefficient.

2. If in the previous example all unregistered sales are concentrated within some interval in the whole period of sales, then $p(0 \mid I, I_r, e) = (I_r - e)/I$, $p(e \mid I, I_r, e) = (I - I_r - e)/I$, and the distribution is uniform for $d \in (0, e)$.

Note that in both cases the expected detected unreported income is $Ed(I, I_r, e) = e(I - I_r)/I$.

3. Consider the case where auditors check reports after the sale is over and cannot distinguish the registered production, but only evaluate the total amount. The accounted production is proportional to the auditors' effort. Then detected unregistered income is $d(I, I_r, e) = 0$ if $e \le I_r$, $e - I_r$ if $e \in (I_r, I)$, $I - I_r$ for $e > I$.

Note. In practice the tax service usually can adjust its effort to the total income and does not apply $e > I$. The results below stay valid for such modification of the model. The standard audit model used in Section 2 is formally equivalent to the case where $e = \pi$, $p(0 \mid I, I_r, \pi) = 1 - \pi$, $p(I - I_r \mid I, I_r, \pi) = \pi$.

The formal setting of the tax enforcement optimization problem for a given detection probability is quite similar to (2.1.1 – 2.1.4). We assume that under a given government strategy $s_G = [T(I_r), e(I_r)]$, each taxpayer aims to maximize her expected income. So, depending on the actual income, her report is

$$I_r(I, s_G) \to \min_{I_r}\{T(I_r) + \sum_d p[d \mid I, I_r, e(I_r)]F(d, I_r)\} \overset{def}{=} T_{eff}(I, s_G).$$

The problem of the government (2.1.2) and the constraint on the expected income of a taxpayer stay the same. The constraint on the minimal after-tax income is now $T[I_r(I)] + F[d, I_r(I)] \le I - I_{\min}$ for any $d \in [I_r, I]$.

The variants of penalty schemes are as follows:

a) $F(d, I_r) = (1 + \delta)[T(I_r + d) - T(I_r)]$;

b) $F(d, I_r) = T(I_r + d) - T(I_r) + \delta d$;

c) $0 \le F(d, I_r) \le I_r + d - T(I_r) - \hat{I}$ for any $d \in [I_r, I]$,

where $\hat{I}$ is a minimal income under the non-optimal behavior of a taxpayer;

d) $F(d, I_r) = \delta d$.

3.2. On optimality of evasion-proof contracts

Now our purpose is to find out under what conditions it is possible to generalize Theorem 2.1.1 for the given model.

Theorem 3.2.1. If the detection probability function meets condition

$$p(0 \mid I, Y, e) \le p(0 \mid I, I_r, e) \text{ for any } I > Y \ge I_r \text{ and } e, \tag{3.2.1}$$

then under penalty constraint c there exists an evasion-proof optimal contract for the tax optimization problem from Section 3.1. The same proposition is true under penalty constraint d, if this function meets condition

$$Ed(I, Y, e) + Ed(Y, I_r, e) \ge Ed(I, I_r, e) \text{ for any } I > Y \ge I_r. \tag{3.2.2}$$

The scheme of the proof is standard (*cf*. CW, Lemma 3 on p.177). Consider a contract $s_G = (T, e)$ such that $I_r(I, s_G) \ne I$ for some I. Let us define a new contract (T^*, e^*) such that $T^*(I) = T_{eff}(I, s_G)$, $e^*(I) = e[I_r(I, s_G)]$. In Appendix we show that under specified conditions this new contract is evasion-proof and equivalent to the original for all agents including the government.

Let us discuss conditions (3.2.1, 3.2.2). Both of them hold for the standard audit model and, moreover, for any model where $p(d \mid I, I_r, e)$ does not depend on I_r. The second condition is also true for examples 1, 2, 3, and seems to be rather general. However, none of these examples meets the first condition, so maybe the optimal contract under constraint c is not typically evasion-proof for the general model of audit.

3.3. The general contract approach

Consider a model given by the distribution of income $G(I)$, the cost $C(e)$ of an audit effort, and the probability $p(I_d \mid I, e)$ of detecting income I_d under actual income I and audit effort e. A general contract c_G for this model consists of a set of messages M, a tax function $T(m)$, $m \in M$, an audit function $e(m)$ and a penalty function $F(I_d, m)$. The only difference with the previous model is that message m may have another meaning than the reported income.

Setting of the optimal contract problem is similar to (2.1.1 – 2.1.4). Each taxpayer aims to maximize her expected income. So depending on the actual income, her message is

$$m(I, c_G) \to \min_m \{T(m) + \sum_{I_d} p(I_d \mid I, e(m)) F(I_d, m)\} \overset{def}{=} T_{eff}(I, c_G). \tag{3.3.1}$$

The problem of the government is to maximize tax revenue net of auditing costs

$$\int \{T_{eff}(I, c_G) - C[e(m(I, c_G))]\} dG(I) \to \max_{c_G} \tag{3.3.2}$$

under the following participation constraints:

$$\int [I - T_{eff}(I, c_G)] dG(I) \geq I_{alt} , \tag{3.3.3}$$

that is, the expected income of an agent under her optimal behavior should exceed the alternative income.

The constraint on the minimal after-tax income is now

$$T[m(I)] + F[I_d, m(I)] \leq I - I_{\min} \tag{3.3.4}$$

for any I_d possible under I and $e(m)$,

and an analog of penalty constraint c for the general contract model is

$$0 \leq F(I_d, m) \leq I_d - T(m) - \hat{I} \tag{3.3.5}$$

for any possible m, I_d.

Any government strategy, or simple contract, considered in Sections 2.1 – 3.2 is a particular case of a general contract with the set $M = [I_L, I_H]$. The following result shows that it is possible to limit the opportunities of the government with simple contracts without any loss of revenue.

Theorem 3.3.1. For any general contract c_G there exists an equivalent simple contract s_G such that

1) $e^*[I_r(I, s_G)] = e[m(I, c_G)]$,

2) $T^*_{eff}(I, s_G) = T_{eff}(I, c_G)$ for any I,

and, moreover, for any income I, distribution of the after-tax income is the same for both contracts.

See Appendix for the proof.

Note that under penalty constraint (3.3.5) there exists, according to the Theorem 3.2.1, the optimal simple contract that is evasion-proof. Condition (3.2.1) holds in this case since the detection probability depends only on I and e. Thus, the optimal evasion-proof simple contract provides the maximal revenue in problem (3.3.2 – 3.3.5).

The possibility of introducing any analog of constraints a, b, d, or, more generally, the principle that "the punishment should meet the crime" in the general contract framework has still to be investigated. The given examples show that reporting of income in practice often essentially differs from sending an abstract message. For these reasons the general contract approach is not so useful in investigating tax enforcement problems.

4. CONCLUDING REMARKS

In this paper we have studied the tax optimization problem under tax evasion for a group of taxpayers with random, independent and equally distributed incomes. Our results clarify the role of participation and penalty constraints in determining the optimal tax enforcement strategy.

According to the known Welfare theorem, if the government has complete information on the income distribution, and the taxpayers are risk-neutral, then it is optimal to impose a lump-sum tax, and not to organize an audit. However, if there exists a constraint on the minimal income under the worst state of nature, then it may be optimal to combine a lump-sum tax with other types of taxes. In this case the maximal lump-sum tax is limited, on the one hand, by the value that can undermine a firm's activity under unfortunate production conditions, and on the other hand, by the expected surplus profit before tax with respect to the desirable level of after-tax profit for this group.

Our results show that it is always optimal to set the maximal lump-sum tax under the mentioned constraints. If the surplus value is relatively small, then introduction of any other tax does not increase net revenue. But if the profit distribution is widely dispersed and the cost of an audit is relatively low, then the optimal evasion-proof contract has the following structure: all incomes below some threshold are taxed to the minimum income, and for all higher incomes the tax is flat. The optimal audit strategy is a known "cut-off" rule: reports below the threshold are audited with the minimal probability that makes tax evasion unprofitable and the rest of the reports are not audited. In some sense the optimal contract does not depend on the penalty rule: it does not matter if the penalty is proportional to the evaded tax or the hidden income, or includes both components. Since the tax is equal to the whole income above the minimal level, only the sum of penalty coefficients matters.

In order to find the optimal threshold income, we may start with the lowest level and increase it until the marginal expenses of an audit exceed the marginal revenue or the expected after-tax profit of the agents reaches its desirable level.

Our formal result is similar to the known propositions on the optimality of regressive taxes (see Mirrlees, CW and others). However, note that the expected income is the same for every taxpayer in our case. We just show that it is optimal to praise fortunate agents with higher incomes.

Surprisingly, this result is relevant to the practice of taxation in Russia. A typical approach is that, for any tax period, the tax service establishes normative tax levels for firms under its supervision proceeding from ex ante information on these firms and some general characteristics of the market in this period (see Metodika, 1997). Such a tax level corresponds to the threshold tax in our model. Only firms that do not pay this normative tax are carefully audited and penalized. One essential difference from our model is that the tax (as well as the penalty) may have another form and the proceeds are not included in the government revenue, but in another fund (for instance, a "voluntary" donation that contributes to some funds of the local administration, see Yakovlev, 2000). Note that criminal groups use a similar approach to enterprises under their control. But this is another story.

One difficulty related to this approach is how to determine desirable after-tax incomes for different groups of taxpayers. From the theoretical point of view, these values should proceed from the solution of the general welfare optimization problem for the economy. In this context our paper permits exclusion of the tax evasion issue from this general problem since we have determined the minimal cost of tax collection for a given desirable income.

An important issue is whether the specified contract is optimal in general, that is, the optimal tax policy of the government is always evasion-proof. In contrast to the conventional point of view (see Chander and Wilde (1998), Mookherjee and Png (1989) and others), we found that under certain conditions the optimal government strategy permits tax evasion if the penalty for evasion is a surcharge on the unpaid tax or the fine is proportional to the hidden income. This happens if there are two possible levels of income, the penalty for evasion is sufficiently soft and the profitability of production is neither very high, nor very low, but exceeds the difference between the minimal value of random income and permissible income (recall Fig. 1).

However, for typical continuous distributions of income (uniform, exponential and others), the optimal evasion-proof contract is optimal in general if the penalty is proportional to the evaded tax.

In this paper we put aside two practically important problems. We consider a group of taxpayers with identically distributed incomes, but actually the government deals with heterogeneous taxpayers. In many cases it has rather incomplete information on their initial characteristics (that is, income distributions), and cannot propose the type-specific contract. We obtained some results for the model of such sort with nonrandom incomes in a complementary paper by Marhuenda, Vasin, and Vasina (2000). However, investigation of the model including both kinds of informational asymmetry remains a challenging task.

Another important issue is corruption in the tax administration. (Recent research on this subject has been done by Hindriks *et al*., 1999.) Let us note that inclusion of a possibility of collusion between a taxpayer and her auditor in our model does not essentially change the formal results on the optimal government strategy. By introducing premiums to auditors for revealing tax evasion, the government can eliminate incentives for accepting bribes and obtain the same optimal tax revenue (see Vasin and Panova, 1999). Kofman and Lawarree (1993, 1996) derive a similar result for the Principal-Supervisor-Agent model in another context. The role of giving incentives to the state officials in fighting corruption was also recognized within the economic literature (see Bardhan, 1977). The actual problem is that the condition of random interaction between taxpayers and auditors in the mentioned models is in contradiction with the reality in many cases. For long-time relations between a taxpayer and an auditor, the mentioned scheme of premiums is not so efficient. So, this problem needs additional investigation.

APPENDIX

Proof of Proposition 2.2.1. Proceeding from Proposition 2.1.2, it suffices to study the case $I_{alt} - I_{\min} < q\Delta I$. Let $y = pF$ denote an actual expected payment to the budget from the additional income. The tax optimization problem in area II can be set as

$$T_L + qy - pc \to \max \tag{A.1}$$

under

$$T_L + qy \le \Delta EI,\ T_L \le T_{LM},\ T_L + \frac{y}{p} \le T_{LM} + \Delta I.$$

Under any permissible $T_L, y > 0$, the optimal p turns the latter relation into equity. Thus, we should maximize

$$R(T_L, y) \overset{def}{=} T_L + qy - cy/(T_{LM} - T_L + \Delta I)$$

in the area represented in Fig. 5.

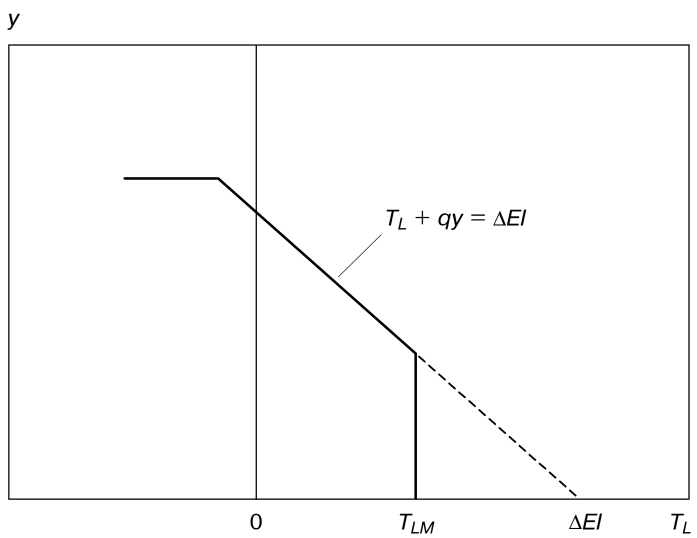

Fig. 5.

Condition (2.2.5) implies that $R(T_L, y)$ increases in y under any permissible T_L. On the line $T_L + qy = \Delta EI$, the revenue is $\Delta EI - cp(y)$, where

$$p(y) = \frac{y}{\Delta I + T_{LM} - \Delta EI + qy}.$$

The audit probability increases in y and reaches its minimum at

$$T_L = T_{LM},\ y = \frac{(\Delta EI - T_{LM})}{q}.$$

Thus, the maximal revenue is

$$R^{II} = \Delta EI - c\frac{\Delta EI - T_{LM}}{\Delta Iq},\ F = \Delta I.$$

In order to find the optimal government strategy, it suffices to compare R^{II} with the maximal revenues in areas I and III. In the latter area $R^{III} = T_{LM}$ since $\Delta T = 0$. Thus, $R^{II} > R^{III}$ because of (2.2.5).

In area I,

$$R^{I} = \Delta EI - (1-q)\frac{c}{1+\delta}$$

and $R^{II} > R^{I}$ if

$$\frac{\Delta EI - T_{LM}}{\Delta Iq} < \frac{1-q}{1+\delta},$$

that is, equivalent to

$$I_{alt} - I_{\min} > q\Delta I\left(1 - \frac{1-q}{1+\delta}\right).$$

If $q\Delta I \le c$, then $R^{III} \ge R^{II}$, and $R^{III} \ge R^{I}$ if $I_{alt} - I_{\min} \ge q\Delta I - c(1-q)p_a^*$.

Proof of Proposition 2.2.2. In area II, setting and solving the tax optimization problem is the same as under a):

$$F = \Delta I,\ p^{II} = \frac{\Delta EI - T_{LM}}{q\Delta I} = \frac{q\Delta I - I_{alt} + I_{\min}}{q\Delta I};$$

the only difference is that $\Delta T = \Delta I(1-\delta_b)$ in this case. In area I,

$$\Delta EI > T_{LM} \Rightarrow\ R^{I} = \max[T_L + q\Delta T - (1-q)cp]$$

under constraints

$$p(\Delta T + \delta\Delta I) \geq \Delta T \ , \ T_L \leq T_{LM}, \ T_L + q\Delta T \leq \Delta EI, \ \Delta T \leq \Delta I \ .$$

Then

$$p^I = \frac{\Delta T}{\Delta T + \delta\Delta I} \ ,$$

and net revenue reaches its maximum when $T_L + q\Delta T = \Delta EI$, $T_L = T_{LM}$,

$$\Delta T = \Delta I - \frac{I_{alt} - I_{\min}}{q} \ .$$

As well as under a), $R^{III} = T_{LM}$, and $R^{II} > R^{III}$ if $q\Delta I > c$. On the other hand, $R^{II} > R^I$ if

$$p^{II} = \frac{\Delta EI - T_{LM}}{q\Delta I} < p^I(1-q) \ .$$

$$\Delta EI - T_{LM} + \delta_b q\Delta I < q\Delta I(1-q) \Leftrightarrow (q + \delta_b)q\Delta I < I_{alt} - I_{\min} < q\Delta I \ .$$

If $q\Delta I \leq c$ then $R^{III} > R^{II}$, and $R^{III} > R^I$ if

$$T_{LM} \geq \Delta EI - c(1-q)\frac{\Delta EI - T_{LM}}{\Delta EI - T_{LM} + q\delta_b\Delta I} \Leftrightarrow$$

$$\Leftrightarrow c(1+q) \geq q\Delta I(1+\delta_b) - (I_{alt} - I_{\min}) \Leftrightarrow$$

$$\Leftrightarrow q\Delta I \geq I_{alt} - I_{\min} \geq q\Delta I(1+\delta_b) - c(1-q) \ .$$

Proof of Proposition 2.2.3 In area I, we aim to maximize $R^I = T_L + q\Delta T - (1-q)cp$ under constraints

$$F + T_L \leq I_H - \hat{I} \ , \ T_L \leq T_{LM}, \ T_L + q\Delta T \leq \Delta EI \ \text{and} \ pF \geq \Delta T \ .$$

Thus, the optimal

$$p^* = \frac{\Delta T}{F} \ , \ F^*(T_L) = I_H - \hat{I} - T_L \ .$$

If $qF_c > (1-q)c$ for any $T_L \leq T_{LM}$, then

$$q - \frac{(1-q)c}{F^*(T_L)} > 0$$

for any $T_L \le T_{LM}$, $T_L^* + q\Delta T^* = \Delta EI$ and the optimal T_L^* minimizes the audit probability proportional to

$$\frac{\Delta EI - T_L}{I_H - \hat{I} - T_L}.$$

Since $I_{alt} > I_{\min} > \hat{I}$, $\Delta EI = I_L + q\Delta I - I_{alt} < I_H - \hat{I}$, and $T_L^* = T_{LM}$.

If $qF_c \le (1-q)c$ then $\Delta T^* = 0$, $T_L^* = T_{LM}$.

Proof of Proposition 2.2.4 is similar to the previous one. The only difference is that $F_d^* = \delta_d \Delta I$, so $q\Delta T^* + T_L^* = \Delta EI$ if $qF_d > (1-q)c$.

Proof of Theorem 2.3.2. According to Proposition 2.3.1, we can express the net revenue under the optimal audit strategy as follows:

$$\int \{T_+^{'}(I)[\mu(I) - (kT_+^{'}(I) + l)^{-1}]\} dG(I).$$

Since the hazardous rate and $T_+^{'}$ decrease in I, the coefficient after $T_+^{'}$ monotonously decreases in I. Thus, under the optimal T, the marginal tax rate $T_+^{'}$ is maximal, that is, equal to 1, till some $\bar{I}$, and is minimal, that is equal to 0 for incomes greater than $\bar{I}$.

Let $R(\bar{I})$ (respectively $R_g(\bar{I})$) denote net (respectively gross) revenue under the tax schedule with threshold $\bar{I}$. Then

$$R(\bar{I}) = \int_{I_L}^{\bar{I}} \rho(I)(I - \frac{c}{k+l})dI + [1 - G(\bar{I})]\bar{I},$$

$$R'(\bar{I}) = \rho(\bar{I})\left(\bar{I} - \frac{c}{k+l}\right) - \rho(\bar{I})\bar{I} + 1 - G(\bar{I}).$$

If

$$\mu(I_L) > \frac{c}{k+l}$$

then $R(\bar{I})$ is a unimodal function with a unique maximum at point $\tilde{I}$ such that

$$\mu(\tilde{I}) > \frac{c}{k+l}.$$

The optimal threshold I^* is equal to $\tilde{I}$ if $R_g(\tilde{I}) \le \Delta EI$, otherwise $R_g(I^*) = \Delta EI$. Under the opposite inequality

$$\mu(I_L) \le \frac{c}{k + l},$$

lump-sum tax $T \equiv T_{LM}$ is optimal.

Now consider the case where $l = 0$, $k = 1 + \delta_a$. In this case condition (2.1.4) takes the form

$$T(I) + \delta_a\{T(I) - T[I_r(I)]\} \le I - I_{\min} \text{ if } q(I) > 0 \quad \text{(a)}$$

$$\text{and } T[I_r(I)] \le I - I_{\min} \text{ if } q(I) < 1. \quad \text{(b)}$$

Here $q(I) = p[I_r(I)]$ is an effective probability of an audit for taxpayers with income I.

Below we prove the theorem under condition

$$T(I) \le I - I_{\min} \text{ for any } I. \quad \text{(A.2)}$$

If $q(I) > 0$ then (a) implies (A.2). We conjecture that this condition is not binding if $q(I) = 0$ and $I_r(I) \ne I$. In any case such constraint seems to be unrestrictive in our setting.

First consider the problem under (A.2) but without condition (2.1.4).

Let us study the optimization problem with respect to an audit rule under a fixed non-decreasing tax schedule T. According to Sanchez and Sobel (1993), the optimal audit rule is a solution to the problem

$$R(q(.)) = \int q(I)[(1 + \delta_a)T_+^{'}(I)\mu(I) - c]dG(I) \to \max_{q(.)}$$

under the following constraints: $q(I)$ does not increase in I,

$$q(I) \in [0, (1 + \delta_a)^{-1}],$$

$$R_g(q(.)) = \int q(I)(1 + \delta_a)T_+^{'}(I)\mu(I)dG(I) \le \Delta EI. \quad \text{(A.3)}$$

Proposition 2 in Sanchez and Sobel shows that there always exists a solution of the form:

$$q^*(I) = \begin{cases} 1/(1+\delta_a) & \text{if } I \in [I_L, I_1), \\ p & \text{if } I \in [I_1, \hat{I}), \\ 0 & \text{if } I \in [\hat{I}, I_H], \end{cases}$$

where $I_L \le I_1 \le \hat{I} \le I_H$, $p \in \left(0, \dfrac{1}{1+\delta_a}\right)$.

This effective probability is generated by an audit rule

$$p^*(I) = \begin{cases} \dfrac{1}{1+\delta_a} & \text{if } I \in [I_L, I_1), \\ p & \text{if } I \in [I_1, I_2), \\ 0 & \text{if } I \ge I_2, \end{cases}$$

where I_2 meets equation $T(I_2) = T_{eff}(\hat{I}) = T(I_1) + p(1+\delta_a)[T(\hat{I}) - T(I_1)]$.

This result easily follows from the linear programming theory: (A.3) may be considered as a linear programming problem with two constraints besides monotonicity of $q(I)$. Hence there exists solution $q^*(I)$ and corresponding $p^*(I)$ which has at most two jumps. Moreover, if $T'_+(I)\mu(I)$ does not increase, then there exists a solution with only one jump, that is a cut-off rule. Note that constraint (2.1.4) is not binding with respect to T that meets (A.2) under any cut-off rule. Moreover, for any pair $T(.)$, $p(.)$ where $p(.)$ is a cut-off rule with a threshold $\bar{I}$, there exist an equivalent evasion-proof strategy $(\bar{T}(.), p(.))$ where

$$\bar{T}(I) = \begin{cases} T(I), & I \le \bar{I}, \\ T(\bar{I}), & I > \bar{I}. \end{cases}$$

In order to complete the proof, let us show that for any permissible pair (T, q), including a 3-level effective audit probability in problem (2.1.2 – 2.1.3, A.2), there exists permissible strategy (T^*, q^*) where, for any I, $T^*(I) = I - I_{\min}$ (so T is a concave tax schedule), q^* is either a 3-level or a cut-off audit rule and $R(T^*, q^*) \ge R(T, q)$.

Consider the problem with respect to T under fixed q. We can rewrite it as follows:

$$R(T,q) = \int T_{eff}(I)dG(I) - c\int_{I_L}^{\hat{I}} q(I)dG(I) \to \max_{T(.)}$$

where

$$T_{eff}(I) = [1 - p(1+\delta_a)]T(I_1) + p(1+\delta_a)T(I) \text{ for } I \in (I_1, \hat{I}], \tag{A.4}$$

$$T_{eff}(I) = T(I) \text{ for } I \le I_1,\ T_{eff}(I) = T_{eff}(\hat{I}) \text{ for } I > \hat{I}$$

under constraints:

$$R_g(T,q) = \int T_{eff}(I)dG(I) \le EI \tag{A.5}$$

and (A.2).

If we exclude constraint (A.5), then T^* is a solution since $T_{eff}(I) \le T^*_{eff}(I)$, proceeding from (A.4, A.2). If (T^*, q) does not meet (A.5) then consider R as a function of $\hat{I}$ under fixed T^*, I_1 and p. For $\hat{I}' \le I_1$ let us define $q(I \mid \hat{I}')$ as a cut-off rule with threshold $\hat{I}'$. Note that R is continuous and increases in $\hat{I}$. Thus there exists $\hat{I}' \in (I_1, \hat{I})$ such that $R_g(T^*, \hat{I}') = \Delta EI$ and $R(T^*, \hat{I}') > R(T, \hat{I})$ because

$$\int q(I \mid \hat{I}')dG(I) = \int_{I_L}^{\hat{I}'} q(I)dG(I) < \int_{I_L}^{\hat{I}} q(I)dG(I).$$

Proof of Theorem 3.2.1. Note that since T is non-decreasing, T^* is also non-decreasing. Let us prove that $I^*_r(I) = I$ under (T^*, e^*). We aim to show that

$$T^*(I,Y) \ge T^*(I) \text{ for any } Y < I. \tag{*}$$

Let

$$I_r = I_r(Y),\ e = e[I_r(Y)] \text{ under } (T,e).$$

Then

$$T^*(I) \le T(I, I_r) = T(I_r) + \sum_d p(d \mid I, I_r, e) F(d, I_r)\ ,$$

$$T^*(I, Y) = T(I_r) + \sum_d p(d \mid Y, I_r, e) F(d, I_r) + \sum_d p(d \mid I, Y, e) p(d, Y)\ .$$

Thus, under condition d, inequality (*) is equivalent to condition (3.2.2). Under c, the inequality is equivalent to

$$p(0 \mid I, Y, e) p(0 \mid Y, I_r, e) T(I_r) + p(0 \mid I, Y, E) p(d \ne 0 \mid Y, I_r, e)(y - \hat{I}) + \\ + p(d \ne 0 \mid I, Y, e)(I - \hat{I}) \ge P(0, I, I_r, e) T(I_r) + p(d \ne 0 \mid I, I_r, e)(I - \hat{I}).$$

The latter follows from (3.2.1).

For example 1,

$$p(0 \mid I, Y, e) = \left(\frac{Y}{I}\right)^e$$

(for small e); for example 2, this probability is

$$\frac{Y - e}{I}$$

for $e < Y$. For example 3, the latter inequality is equivalent to $Y \ge I$. Thus, in all these cases (3.2.1) is not true in general.

Proof of Theorem 3.3.1. For a given contract c_G, for any message m, let $Inc(m)$ denote a set of incomes such that $m(I, c_G) = m$. For any I_r such that $I_r = I_r(m) \overset{def}{=} \min Inc(m)$, and for some m, let $e^*(I_r) = e(m)$, $T^*(I_r) = T(m)$, $F^*(I_d, I_r) = F(I, m)$. If $I_r \ne I_r(m)$ then let $T^*(I_r) = \min\{T(m)$ for m such that

$$I_r(m) \ge I_r\ \},\ \ F^*(I_d, I_r) = \max_m F(I_d, m)\ ,\ \ e^*(I_r) = e[m(I_L)]\ .$$

Such definition preserves monotonicity of the tax function: if $T[m(I)]$ does not decrease in I, then $T^*(I)$ does not decrease in I. Besides that, if the original penalty function meets condition (3.3.5), then the new function meets

$0 \le F^*(I_d, I_r) \le I_d - T^*(I_r) - \hat{I}$ for any possible I_d, I_r.

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